

Student Workbook

Complex Numbers

Name: _____

Date: _____

Period: _____

Learning Target: I can perform operations with complex numbers and find the modulus of a complex number.

Vocabulary

Term	Meaning
Imaginary Unit (i)	Defined as $i = \sqrt{-1}$, so $i^2 = -1$.
Complex Number	A number of the form $a + bi$, where a is the real part and b is the imaginary part.
Complex Conjugate	For $a + bi$, the conjugate is $a - bi$. Used to simplify division.
Modulus ($ z $)	The distance from a complex number to the origin on the complex plane: $ a+bi = \sqrt{a^2+b^2}$.

Key Concept: Powers of i

THE CYCLE OF 4

$i^1=i, i^2=-1, i^3=-i, i^4=1$, then it repeats. To simplify i^n , divide n by 4 and use the remainder.

EXAMPLE 1

Simplify i_{15} .

$15 \div 4 = 3$ remainder _____ $\rightarrow i_{15} = i_{\text{_____}} = \text{_____}$

Key Concept: Operations with Complex Numbers

EXAMPLE 2 — Add, Subtract, Multiply

$(3+2i)+(1-5i) = \text{_____}$

$(3+2i)(1-5i) = 3-15i+2i-10i^2 = 3-13i+10 = \text{_____}$

COMMON MISTAKE

When multiplying, don't forget to replace i^2 with -1 ! Leaving i^2 in your answer (or treating it as $+1$) is the most common error with complex numbers.

Key Concept: Dividing Complex Numbers

METHOD

Multiply the numerator and denominator by the conjugate of the denominator. This eliminates i from the bottom.

EXAMPLE 3

Divide $(4+3i) / (2-i)$.

$$\begin{aligned} \text{Multiply by } (2+i)/(2+i): (4+3i)(2+i) / (2-i)(2+i) &= (8+4i+6i+3i^2) / (4+1) \\ &= (8+10i-3) / 5 = \underline{\hspace{2cm}} \end{aligned}$$

Key Concept: The Modulus & the Complex Plane

FORMULA

$|a+bi| = \sqrt{a^2+b^2}$ — this is just the Pythagorean theorem! The complex number $a+bi$ is plotted as the point (a,b) , with a on the horizontal (real) axis and b on the vertical (imaginary) axis.

EXAMPLE 4

Find the modulus of $3-4i$.

$$|3-4i| = \sqrt{(3^2+(-4)^2)} = \sqrt{(9+16)} = \sqrt{\hspace{1cm}} = \underline{\hspace{1cm}}$$

Guided Practice — Part A: Simplify Powers of i

A1 i_{11}

A2 i_{28}

A3 i_{37}

Guided Practice — Part B: Add, Subtract, Multiply

B1 $(5-3i)+(2+7i)$

B2 $(6+i)-(4-2i)$

B3 $(2+4i)(3-i)$

Guided Practice — Part C: Divide

C1 $(6+2i) / (1-i)$

C2 $(3-i) / (2+3i)$

Guided Practice — Part D: Find the Modulus

D1 $|6+8i|$

D2 $|-5+12i|$

BEFORE YOU MOVE ON

Check with a partner: did you substitute -1 for every i^2 in your multiplication and division work?